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A GRAPHITE RESISTANCE HEATER FOR A HYPERSONIC WIND TUNNEL USING NITROGEN: PART II. ANALYSIS OF HEATER PERFORMANCE

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Abstract—A formula is derived for the rise in total temperature of nitrogen flowing through an electric resistance element made of graphite. The derivation of the formula involves, firstly, the consideration of an idealized heating system based on a simple graphite tube, and, secondly, the assumption that the solution of the internal heat transfer problem possesses certain properties. The formula provides a useful basis for the analysis of experimental results from practical heating systems which resemble the idealized system. Such a practical system is exemplified by the graphite heater in the pilot hypersonic nitrogen tunnel at Princeton University (as described in Part I), and the results obtained with this heater are shown to be well represented by the formula.

NOMENCLATURE

a, constant used in analytical representation of enthalpy of gas;
b, constant used in analytical representation of resistivity of graphite;
c_{p_0}, specific heat of gas at room temperature;
$f(\theta)$, function involved in description of element temperature distribution;
h_t, specific total enthalpy of gas;
h_0, value of h_t at room temperature;
i, current;
l, length of idealized element;
m, mass flow of gas when current i passing through element;
m_0 mass flow of gas at room temperature;
p_{t_0}, total pressure of gas (controlled to have room temperature value when current i passing);
q, rate of external heat loss from element;
r, resistance of element when current i passing;
r_0, resistance of element at room temperature;
r_{min}, minimum resistance of element with respect to gas total temperature during a test;
x, distance along element measured from gas entrance;
A_E, cross-sectional area of idealized element, excluding area of gas passage;
A_0^*, area of sonic orifice (kept at room temperature value when current i passing);
R, gas constant for unit mass;
T_t, total temperature of gas when current i passing through element;
T_0, room temperature (taken to be 530°R here);
T_E, local temperature of point on element.

Greek symbols
α, thermal efficiency parameter, defined by $\psi = [(1/\alpha) - 1]\tau$;
β, heat transfer parameter, defined by $\tau_{E_{max}} = \tau/\beta$;
θ, dummy variable used in expressing element temperature distribution;
κ_0, thermal conductivity of element material at room temperature;
μ, non-dimensional power variable, defined by $\mu = ri^2/m_0 c_{p_0} T_0$;

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ν, non-dimensional resistance variable, defined by $\nu = r/r_0$;
$\nu_{\min}$, minimum value of ν with respect to τ;
ξ, x/l;
$\xi_{\max}$, value of ξ at which $\tau_E = \tau_{E_{\max}}$;
ρ, resistivity of element material;
ρ_0, value of ρ at room temperature;
σ, non-dimensional current variable, defined by $\sigma = r_0 i^2/m_0 c_{p_0} T_0$;
τ, non-dimensional gas total temperature variable, defined by $\tau = (T_t - T_0)/T_0$;
τ_E, non-dimensional element temperature, defined by $T_t = (T_E - T_0)/T_0$;
$\tau_{E_{\max}}$, maximum value of τ_E attained along element;
ϕ, non-dimensional form of rate of heat addition to gas, defined by $\phi = m(h_t - h_0)/m_0 c_{p_0} T_0$;
ψ, non-dimensional form of external heat loss, defined by $\psi = q/m_0 c_{p_0} T_0$;
Γ, factor involved in expression for mass flow through a sonic orifice.

1. INTRODUCTION

A FORMULA is given for the rise in total temperature of nitrogen flowing through an electric resistance element made of graphite, in terms of the current and the relevant parameters of the heating system. The range of total temperature under consideration is between room temperature (say 530°R) and about 5000°R (see Part I, [1]). At gas temperatures above 5000°R, corresponding maximum graphite temperatures may approach the sublimation limit, so that this temperature range is near to the full range of practical interest for graphite and nitrogen.

The derivation of the formula is given in Section 2. The essential steps of the derivation are, firstly, the specification of an idealized heating system which permits many theoretical simplifications, and secondly, the assumption that the solution of the internal heat transfer problem possesses several simplifying properties. Since the desired formula for gas total temperature involves only bulk quantities, it is not too sensitive to the various idealizations and assumptions made, and it is readily amenable to empirical interpretation for application to a practical heating system. This interpretation is discussed in Section 3, where experimental results obtained with the graphite heater in the pilot hypersonic nitrogen tunnel at Princeton University [1] are presented and shown to be consistent with the feature of the formula. The results quoted are for one value of gas total pressure only, and some remarks on the possibilities of the formula for predicting heater performance at other pressures are made in Section 4.

2. THEORY FOR AN IDEALIZED HEATING SYSTEM

2.1. *Simplifications afforded by the idealized system*

The following idealized heating system is considered. The heating element is constructed from a pure impermeable material in the form of a straight thin-walled cylindrical tube of circular cross section, of length l and cross sectional area A_E (excluding the area of the gas passage). At room temperature T_0 there is an adiabatic forced-convection flow of a pure inert thermally perfect gas through the element; this is called the cold flow. The mass flow of gas, m_0 in the cold flow, is controlled by a sonic orifice of area A^* external to the element. An electric current i is then passed through the element. The total temperature of the gas at the entrance to the element is maintained at room temperature T_0 while the current is passing. The ends of the element and the orifice are maintained, by suitable cooling, at room temperature T_0 also. The gas total temperature T_t and the gas total pressure do not vary between the exit of the element and the orifice. The gas total pressure at the orifice is controlled to be constant at its value p_{t_0} in the cold flow. The cooling at the orifice is assumed to keep the area of the orifice constant at the value A_0^* while the current is passing. The element is surrounded by a perfectly efficient radiation shield.

It is required to find a relation for the total temperature T_t of the gas at the exit of the element in terms of the applied current i and the relevant cold flow parameters of the element geometry, the element material, the gas, and the external features of the system. Such a relation is regarded as describing the performance of the heating system under fixed initial (that is, cold flow) conditions. The effects of changes in the cold flow parameters, for instance in the gas total pressure, are considered as a separate problem at this stage and are not necessarily sought now.

It is convenient to represent the current i by the non-dimensional variable σ defined by

$$\sigma = \frac{r_0 i^2}{m_0 c_{p_0} T_0} \tag{1}$$

where r_0, m_0 and c_{p_0} are the (cold flow) values of the element resistance, the gas mass flow and the gas specific heat taken at room temperature T_0. The gas total temperature T_t is represented by the non-dimensional variable τ defined by

$$\tau = \frac{T_t - T_0}{T_0}. \tag{2}$$

In principle i is the independent variable and T_t the dependent variable, but for analytical purposes it happens that, rather than seek a relation for τ in terms of σ, it is better to consider σ as a function of τ. The range of τ of interest is taken to be $0 \leqslant \tau \leqslant 9$.

In seeking a formula for σ in terms of τ it is advantageous to write

$$\sigma = \frac{\mu}{\nu}, \tag{3}$$

where μ and ν are defined, in terms of the resistance r of the element when the current i is passing, by

$$\mu = \frac{r i^2}{m_0 c_{p_0} T_0}, \tag{4}$$

$$\nu = \frac{r}{r_0}. \tag{5}$$

The aim is now to find the non-dimensional power variable μ and the non-dimensional resistance variable ν in terms of τ.

The external energy balance states that the electrical power supplied to the element is equal to the sum of the rate of heat addition to the gas and the rate of external heat loss. Hence

$$r i^2 = m(h_t - h_0) + q, \tag{6}$$

where m is the gas mass flow when the current i is passing through the element and $(h_t - h_0)$ is the increase of the specific total enthalpy of the gas from the entrance to the exit of the element; q is the rate of external heat loss. The energy balance may now be written in the non-dimensional form

$$\mu = \phi + \psi, \tag{7}$$

where ϕ and ψ are given by

$$\phi = \frac{m(h_t - h_0)}{m_0 c_{p_0} T_0}, \tag{8}$$

$$\psi = \frac{q}{m_0 c_{p_0} T_0}. \tag{9}$$

Since the mass flow m through the system is governed by the external sonic orifice, it can be expressed as

$$m = \frac{\Gamma A_0^* p_{t_0}}{(R T_t)^{\frac{1}{2}}}, \tag{10}$$

where R is the gas constant for unit mass and Γ is a factor which is constant for a perfect gas with constant specific heats and which for a thermally perfect gas depends on T_t only. Now a study of the properties of nitrogen [2] show that, in the range of temperatures under consideration, Γ is numerically constant to within about 1 per cent. Therefore, Γ is assumed here to be exactly constant and its value is taken to be the value appropriate to the perfect gas, which is also its value in the cold flow. Hence, in the cold flow the gas mass flow is given by

$$m_0 = \frac{\Gamma A_0^* p_{t_0}}{(R T_0)^{\frac{1}{2}}}. \tag{11}$$

Therefore, it follows from equations (10), (11) and (2) that

$$\frac{m}{m_0} = (1 + \tau)^{-\frac{1}{2}}. \tag{12}$$

It may be remarked here that the inversion of this relation gives the gas total temperature in terms of the gas mass flow, and hence provides a means of measuring total temperature indirectly [1]. Now, for a thermally perfect gas the quantity $(h_t - h_0)/c_{p_0} T_0$ is a function of τ only, and for nitrogen in the range of temperatures of interest it can be represented analytically by the expression

$$\frac{(h_t - h_0)}{c_{p_0} T_0} = \tau(1 + a\tau), \tag{13}$$

where a is a small constant which represents the departure of the gas from a calorically perfect gas. Therefore, it follows that for a given gas ϕ depends on τ only, and is given as a function of τ by

$$\phi = (1 + \tau)^{-\frac{1}{2}}\tau(1 + a\tau). \tag{14}$$

The rate of external heat loss q is given, in the absence of losses to the surroundings due to radiation, by the rate at which heat is conducted away from the cooled ends of the element. For a thin-walled element of circular cross section the temperature distribution may be taken to be spatially dependent only on the distance x along the element from the gas entrance, and therefore is given by

$$q = \kappa_0 A_E \left(\left|\frac{dT_E}{dx}\right|_{x=0} + \left|\frac{dT_E}{dx}\right|_{x=1} \right), \tag{15}$$

where κ_0 is the thermal conductivity of the element material at room temperature and $|dT_E/dx|_{x=0}$ and $|dT_E/dx|_{x=1}$ are the moduli of the temperature gradients along the element at the entrance and exit respectively. Since in the idealized heating system the element temperature T_E increases from T_0 at the entrance to a maximum and then decreases back to T_0 at the exit, it is convenient to introduce the non-dimensional quantities τ_E and ξ defined by

$$\tau_E = \frac{T_E - T_0}{T_0}, \tag{16}$$

$$\xi = \frac{x}{l}. \tag{17}$$

Then the expression for ψ becomes

$$\psi = \frac{\kappa_0 A_E}{m_0 c_{p_0} l} \left(\left|\frac{d\tau_E}{d\xi}\right|_{\xi=0} + \left|\frac{d\tau_E}{d\xi}\right|_{\xi=1} \right). \tag{18}$$

It does not seem possible to go further in the evaluation of ψ, and hence of μ, without a consideration of the internal heat transfer problem.

The resistance of the element r depends on the integrated effect of the variation of the resistivity ρ of the element material with the local element temperature T_E. Since the element is thin-walled and cylindrical, and hence of constant cross-section area, it follows that

$$r = \frac{1}{A_E}\int_0^1 \rho \, d\xi. \tag{19}$$

Also, the resistance in the cold flow is

$$r_0 = \frac{l \rho_0}{A_E}, \tag{20}$$

where ρ_0 is the resistivity at room temperature. Hence

$$\nu = \int_0^1 \frac{\rho}{\rho_0} \, d\xi, \tag{21}$$

where ρ/ρ_0 is considered at this stage as a function of local temperature, that is as a function of τ_E. Now the variations of resistivity for several grades of graphite, in particular the rather extreme grades named lampblack-base graphite and petroleum-coke-base graphite, are known [3]. It appears that the form of ρ/ρ_0 for lampblack-base graphite can be fitted with very good accuracy by the expression $\rho/\rho_0 = (1 + \tau_E)^{-\frac{1}{2}}$, while the form of ρ/ρ_0 for petroleum-coke-base graphite can be fitted with equally good accuracy by the expression $\rho/\rho_0 = 0{\cdot}130\, \tau_E + (1 + \tau_E)^{-\frac{1}{2}}$. Therefore, the following generalized expression is taken for the variation of the resistivity of graphite:

$$\frac{\rho}{\rho_0} = b\tau_E + (1 + \tau_E)^{-\frac{1}{2}}, \tag{22}$$

where b is a constant. Hence, it follows that for a graphite element

$$\nu = \int_0^1 [b\tau_E + (1 + \tau_E)^{-\frac{1}{2}}] \, d\xi. \tag{23}$$

Again, this is as far as it seems possible to go without considering the internal heat transfer problem.

2.2. *Assumptions about the solution of the internal heat transfer problem*

Since the interest in this investigation is in bulk quantities rather than in detailed properties of the solution of the internal flow, an attempt to deduce the element temperature distribution is not made, and instead it is assumed to have a simplified form. (It is not asserted that the solution may have the assumed form, only that the assumed form may lead to not implausible forms for ψ and ν.) Firstly, it is assumed that the element temperature distribution possesses a kind of symmetry which is represented by

$$\frac{\tau_E}{\tau_{E\max}} = f(\theta), \left\{ \begin{array}{l} \theta = \dfrac{\xi}{\xi_{\max}}, \quad 0 \leqslant \xi \leqslant \xi_{\max}, \\ \theta = \dfrac{1-\xi}{1-\xi_{\max}}, \\ \qquad \xi_{\max} \leqslant \xi \leqslant 1, \end{array} \right\} \quad (24)$$

where $\tau_{E\max}$ is the maximum value of τ_E, $\xi_{\max}$ is the value of ξ at which $\tau_E = \tau_{E\max}$, and $f(\theta)$ is a function which satisfies the conditions $f(0) = 0$, $f(1) = 1$ and $f'(1) = 0$. Secondly, it is assumed that $f(\theta)$ is a function independent of τ and of the various parameters of the system, and that $\xi_{\max}$ is independent of τ and depends only on the system parameters. The entire dependence of τ_E on τ is then confined to the dependence of $\tau_{E\max}$ on τ, and the third assumption made is that the heat transfer from the element to the gas is represented by the expression

$$\tau_{E\max} = \frac{\tau}{\beta}, \quad (25)$$

where β is independent of τ and depends only on the system parameters.

It then follows that ψ may be written as

$$\psi = \left(\frac{\kappa_0 A_E f'(0)}{m_0 c_{p_0} l \xi_{\max}(1-\xi_{\max})\beta}\right)\tau. \quad (26)$$

Now, since both $\xi_{\max}$ and β are assumed to be independent of τ, then in effect ψ is assumed to be proportional to τ. The geometrical parameters of the element may be eliminated from the constant of proportionality by introducing r_0 and ρ_0, so that an alternative expression for ψ is

$$\psi = \left(\frac{\kappa_0 \rho_0 f'(0)}{m_0 c_{p_0} r_0 \xi_{\max}(1-\xi_{\max})\beta}\right)\tau; \quad (27)$$

a further useful feature of this form is that for graphites $\kappa_0\rho_0$ is effectively independent of the grade of graphite [4]. However, still another form of the constant of proportionality is more convenient, since it is no less general to write

$$\xi_{\max}(1-\xi_{\max}) = \frac{\kappa_0\rho_0 f'(0)}{m_0 c_{p_0} r_0[(1/\alpha)-1]}, \quad (28)$$

and then the expression for ψ becomes

$$\psi = \left(\frac{1}{\alpha} - 1\right)\tau. \quad (29)$$

The constant of proportionality is written in this way so that the parameter α is equal to the limit of the thermal efficiency $m(h_t - h_0)/ri^2 = \phi/(\phi+\psi)$ as τ tends to zero; that is, α is the initial thermal efficiency of the heating system. Therefore, by using equations (7), (14) and (29) an explicit expression for μ in terms of τ is obtained:

$$\mu = (1+\tau)^{-\frac{1}{2}}\tau(1+a\tau) + \left(\frac{1}{\alpha} - 1\right)\tau. \quad (30)$$

The assumed form of solution for the internal problem leads to the following expression for ν:

$$\nu = \int_0^1 \left[b\frac{\tau}{\beta}f(\theta) + \left(1 + \frac{\tau}{\beta}f(\theta)\right)^{-\frac{1}{2}}\right] d\theta; \quad (31)$$

it is noted that ν does not involve $\xi_{\max}$ and therefore may be regarded as independent of α. In order to obtain an explicit expression for ν in terms of τ it is necessary to take one more step, namely to choose a form for $f(\theta)$. The chosen form is the simplest which satisfies the given conditions, that is

$$f(\theta) = \theta(2-\theta). \quad (32)$$

(A form of $f(\theta)$ of more theoretical significance could be chosen here if it were considered desirable and justifiable.) Hence, the expression for ν becomes

$$\nu = \frac{2}{3}b\left(\frac{\tau}{\beta}\right) + \left(\frac{\tau}{\beta}\right)^{-\frac{1}{2}} \sin^{-1}\left\{\left(\frac{\tau}{\beta}\right)^{\frac{1}{2}}\left(1+\frac{\tau}{\beta}\right)^{-\frac{1}{2}}\right\}. \quad (33)$$

It is noted that this expression for ν has a minimum value $\nu_{\min}$ (corresponding to a minimum resistance $r_{\min}$), and that $\nu_{\min}$ depends on b only; it is not possible to express $\nu_{\min}$ explicitly in terms of b, but it is easily calculated numerically.

The final result for the required performance formula connecting σ and τ and the system parameters in the appropriate range of τ may now be written by using equations (3), (30) and (33):

$$\sigma = \left[\frac{(1+\tau)^{-\frac{1}{2}}\tau(1+a\tau) + \left(\frac{1}{\tau} - 1\right)\tau}{\frac{3}{2}b\left(\frac{\tau}{\beta}\right) + \left(\frac{\tau}{\beta}\right)^{-\frac{1}{2}} \sin^{-1}\left\{\left(\frac{\tau}{\beta}\right)^{\frac{1}{2}}\left(1+\frac{\tau}{\beta}\right)^{-\frac{1}{2}}\right\}}\right]. \tag{34}$$

By specifying an idealized problem and then assuming that its solution has certain simplifying features, the heater performance (that is the relation between the variables τ and σ) has been expressed in terms for four parameters a, b, and α, β. Of these parameters, a and b are properties of the gas and the element material respectively and are constant with respect to τ. However, the thermal efficiency parameter α and the heat transfer parameter β are not strict constants but are constants only in consequence of the assumptions made in the derivation of the performance formula. Moreover, it is not known how α and β depend on the basic cold flow parameters of the system. Therefore, it is preferable to regard the arguments given here as serving merely to suggest possible significant forms for μ and ν. It is then necessary to seek experimental evidence concerning μ and ν. However, the idealized system itself is an impractical one because an element long enough to give adequate heat transfer would be too long structurally, and therefore evidence must be sought from a practical system which bears some resemblance to the idealized system.

3. ANALYSIS OF EXPERIMENTAL RESULTS FROM THE PRINCETON UNIVERSITY GRAPHITE HEATER

A practical heating system which does resemble the idealized system is that in use in the pilot hypersonic nitrogen tunnel of the Gas Dynamics Laboratory at Princeton University. Full details of the tunnel and the heating system are given in Part I [1], and it is sufficient here to note how the practical system differs from the idealized one previously considered. The principal difference is concerned with the element geometry: the practical element contains a spiral gas passage instead of a straight-through one, and the gas passage is of rectangular and not circular cross-section; also, the area of the gas passage is not constant near the entrance and exit of the element nor is the cross-sectional area of the element itself constant in these regions. The other main difference is that the end conditions on gas total temperature and element temperature assumed in the idealized system are probably not precisely reproduced in the practical system, since the cooling systems applied to the ends of the element and to the orifice may not be sufficiently effective to reduce the temperatures there exactly to room temperature.

During the time that the pilot nitrogen tunnel has been in operation a large number of tests have been made, using nitrogen of different degrees of purity and elements of various lengths and cross-sectional areas, constructed from various grades of graphite, some composed of the same grade of graphite throughout and others coated with a thin layer of pyrolytic graphite, and with different gas mass flows determined by the gas total pressure selected. In this paper, although some remarks based on general experience are made, only a sample of results is presented in detail. These refer to some of the tests in which super-high-purity nitrogen was used, the elements were coated with pyrolytic graphite and were nominally identical (having a length of about 8 in and an area of about $\frac{1}{8}$ in^2 with a spiral gas passage about 27 in long and of about $\frac{1}{12}$ in equivalent diameter), and the gas total pressure was fixed at 1000 lbf/in^2 (thus giving a cold mass flow of about 0·018 lb/s at room temperature). Results are presented for four distinct elements designated here as elements 1, 2, 3, 4. Each element was tested several times. The results for the first test of each element are given, together with results from four subsequent tests of element 2, namely the fifth, tenth, fifteenth and twentieth. This selection of results was made to illustrate the trends of the experiments with as little detail as possible. The results obtained in all the tests of other elements, and in other runs of the elements considered here, are in general agreement with the results presented in this paper.

The results presented are for the power variable μ and the resistance variable ν in terms of the gas total temperature variable τ (Figs. 1 and 2 respectively). The heater performance curve of τ against the current variable σ is shown in Fig. 5 of Part I.

The derivation of μ requires the measurement of the cold mass flow m_0 (the value of T_0 is taken as 530°R throughout the c_{p_0} is known) and the measurement of the current, the resistance and the gas total temperature in the hot flow. The values of m_0 in the tests considered are listed in Table 1 of Part I. The resistance was obtained from the measurement of the voltage across the ends of the element, and the gas total temperature was obtained from the measurement of the gas mass flow in the hot flow [1]. It is seen from Fig. 1 that μ is well represented by a formula of the type of equation (30). The value of a for nitrogen is taken to be 0·019 and the value of $\alpha = 0{\cdot}79$ was obtained as the mean of the individual values obtained from each experimental point (α can be calculated directly from equation (30) when μ, a and τ are known). The value of α is constant to within ± 2 per cent. It may be mentioned that in all the tests performed in the pilot nitrogen tunnel with one notable exception, the evidence has indicated the constancy of α over the full range of τ. The exception occurred when the radiation shield was not installed in the stagnation chamber, in which case α decreased by more than 10 per cent between $\tau = 1$ and $\tau = 6$. It must be noted that the excellence of the correspondence between the experimental results and the theoretical formula is contributed to by both the method of measurement of the gas total temperature (which uses equation (12) which is involved in the theory also) and the use of an empirically derived value for α.

The derivation of ν requires the measurement of the resistance of the element in the cold flow r_0 and of the resistance and the gas total temperature in the hot flow. However, it was found in these experiments in the pilot nitrogen tunnel that it was not possible to measure r_0 with adequate accuracy because of contact resistances, but this inability to actually measure r_0 turned out to be an advantage from the point of view of the analysis of the results. It was found that the resistance of an element during a test had a minimum value $r_{\min}$ [as predicted by equation (33)] which occurred within the range of gas temperatures under investigation, and this fact, combined with the result that ν as given by equation (33) has a minimum value $\nu_{\min}$ which is a (numerically) known function of b, enables a simple procedure to be adopted for calculating r_0 when b is known, namely by deducing $r_{\min}$ and evaluating r_0 as $r_{\min}/\nu_{\min}$. However, it also happened in this work that the appropriate value of b for the graphite used was not known at the outset, but the fact that the theoretical result for $\nu_{\min}$ depended in a known way on b only was used to determine b as follows: the value of $r_{\min}$ for each test of each element was deduced; then the resulting values of $r/r_{\min}$ for each test of each element were plotted on the same graph and the extrapolation of the results down to $\tau = 0$ was then made; this yielded a value of $1/\nu_{\min}$ and hence in turn yielded the value of b; the result obtained was $b = 0{\cdot}025$. (This procedure assumes in effect that any differences between tests of the nominally identical elements are concentrated in the value of r_0.) The resulting values of r_0 are listed in Table 1 of Part I. It is seen from Fig. 2 that the experimental values of ν are well represented by a formula of the type of equation (33). The empirical curve shown uses a mean value of $\beta = 0{\cdot}75$ obtained from the individual values of β corresponding to each experimental point. (A numerical procedure is needed to calculate β from equation (33) when ν, b and τ are known.) These individual values do not indicate any definite variation of β and τ but the scatter about the mean value is high, being of the order of ± 20 per cent. The correspondence of the experimental results and the theoretical formula is good, but it must be noted that the use of a basic feature of the formula to deduce both b and r_0 contributes greatly to the agreement, as also does the fact that an empirically derived value of β was used.

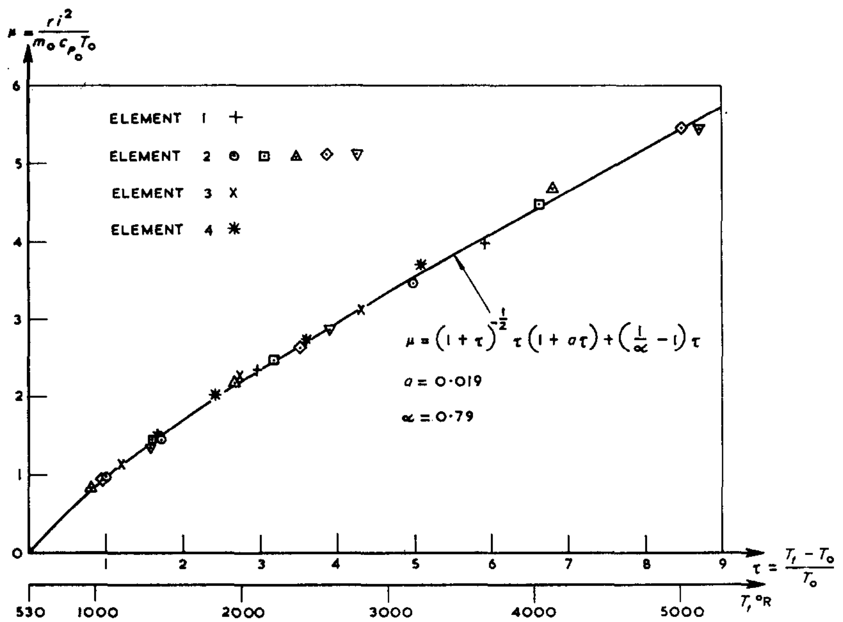

FIG. 1. Variation of power variable μ with gas total temperature variable τ.

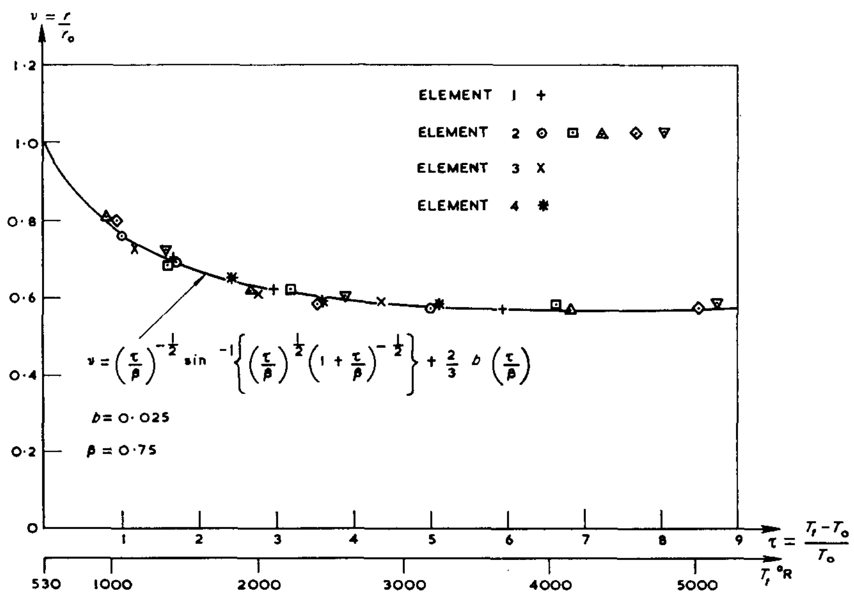

FIG. 2. Variation of resistance variable ν with gas total temperature variable τ.

Some independent evidence which helps to support the notion of the effective constancy of α and β during a test, and is consistent with the particular values obtained in this analysis, is provided by some early experiments on uncoated elements in which measurements of the element temperatures were made by tungsten–rhenium thermocouples. It was found that, for the values of τ up to about 3, there was only a small variation in $\tau/\tau_{E_{max}} = \beta$, and the mean value of β was about 0·75. Moreover, it was demonstrated that the maximum element temperature was located close to the gas exit of the element, an observation which is consistent with the estimate $\xi_{max} = 0{\cdot}94$ obtained from equation (28) by taking $\kappa_0 \rho_0 = 0{\cdot}0013$ WΩ/degR [4], $f'(0) = 2$ [from equation (32)], $m = 0{\cdot}018$ lb/s (see Table 1 of Part I), $c_{p_0} = 260$ J/lb degR [2], $r_0 = 0{\cdot}050$ Ω (see Table 1 of Part I) and $\alpha = 0{\cdot}79$, $\beta = 0{\cdot}75$. Consequently, it seems that the concept of two empirical parameters α and β which typify the performance of a heating system is a useful one for the purpose of analysing experiment results. However, no more than this is claimed, and further tests of greater refinement may well indicate definite trends of α and β with change of τ (especially variations in β) which conform to the predictions of a more detailed theory than that attempted here.

The result that $\beta = 0{\cdot}75$ implies that when the gas stagnation temperature is 5000°R the maximum element temperature is about 6500°R, so that it appears that the graphite element in this situation is operating near to its sublimation limit of about 7000°R. It is concluded that at the stagnation pressure of 1000 lbf/in² the graphite heater is nearing its full potential. In order to satisfy the full heating requirements for a hypersonic wind tunnel, however, the graphite heater must achieve its full performance at higher pressures, of the order of 10 000 lbf/in². There seems no reason why the effects of high pressure should not be included in a modified form of the crude theory given here, which would involve still the two unknown empirical parameters α and β. A possible means of characterizing the heater performance at higher pressures would then be by establishing the variations of α and β with stagnation pressure. The pilot hypersonic nitrogen tunnel at Princeton University is being used to extend the range of performance of the graphite heater to higher pressures.

4. CONCLUDING REMARKS

A heuristic theoretical analysis has indicated an explicit expression for the variation of the total temperature of a gas flowing through an electric resistance element with the current applied to the element. The expression involves two unknown parameters, the thermal efficiency parameter α and the heat transfer parameter β, which are assumed to depend only on the cold-flow parameters of the heating system. In the absence of expressions for α and β in terms of the cold-flow parameters, they must be determined experimentally. An analysis of experimental results obtained, in a series of tests at a fixed gas total pressure of 1000 lbf/in², with the heating system in the pilot hypersonic nitrogen tunnel at Princeton University has indicated that α and β were substantially constant during these tests and has provided their appropriate mean values for the particular pressure. However, for the most effective use of hypersonic wind tunnels it is required to operate at higher pressures, of the order of 10 000 lbf/in². It is suggested that the performance formula derived here may be helpful in extending the range of application of electric resistance heaters to higher pressures by enabling attention to be concentrated on the variation with pressure of the two empirical parameters α and β.

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Résumé—On établit ici une formule donnant l'élévation de température totale de l'azote s'écoulant à travers une résistance électrique en graphite. Pour ceci on considère premièrement un système de chauffage idéalisé constitué par un simple tube de graphite, et deuxièmement l'hypothèse que la solution du problème de transport de chaleur interne possède certaines propriétés. La formule fournit une base utile pour l'analyse des résultats expérimentaux obtenus avec les systèmes de chauffage réels qui ressemblent au système idéalisé. Un système de chauffage de ce genre est concrétisé par le réchauffeur en graphite de la soufflerie pilote hypersonique de l'Université de Princeton (décrite dans la partie I) et les résultats obtenus avec ce réchauffeur sont bien représentés par la formule.

Zusammenfassung—Stickstoff strömt durch ein Widerstands-Heizelement aus Graphit. Eine für diesen Fall geltende Gleichung zur Errechnung des Temperaturanstiegs wird abgeleitet. Dabei wird der idealisierte Fall eines einfachen Graphitrohres zu Grunde gelegt und ausserdem vorausgesetzt, dass die Lösung des inneren Wärmeübergangs gewisse Eigenschaften besitzt. Die Gleichung eignet sich zur Interpretation von Messergebnissen an ausgeführten Erhitzern, wenn diese nicht sehr von dem idealisierten Fall abweichen. Dies gilt für den Graphiterhitzer des stickstoffbetriebenen Hyperschall Windkanals der Princeton University (s. Beschreibung im Teil I) und die daran gewonnenen Messergebnisse stimmen gut mit der Gleichung überein.

Аннотация—Выводится формула, характеризующая нагревание потока азота, протекающего через элемент электрического сопротивления из графита. При выводе формулы сначала проводится анализ идеализированной системы, состоящей из простой графитовой трубки, а затем принимается допущение, что решение задачи внутреннего теплообмена характеризуется определенными свойствами. Эта формула используется для обработки экспериментальных данных, полученных в результате практической работы систем нагрева, близких к идеализированной. Примером такой системы является графитовый нагреватель в гиперзвуковой аэродинамической трубе полупромышленного типа с потоком азота, использующейся в Принсетонском Университете (Описание приводится в ч.I). Полученные результаты подтверждаются этой формулой.